NSW Education Standards Authority

2022 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number on the Question 12 Writing Booklet attached

Total marks: 70

Section I – 10 marks (pages 2–9)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 10–18)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 It is given that $\cos\left(\dfrac{23\pi}{12}\right) = \dfrac{\sqrt{6}+\sqrt{2}}{4}$.

Which of the following is the value of $\cos^{-1}\left(\dfrac{\sqrt{6}+\sqrt{2}}{4}\right)$?

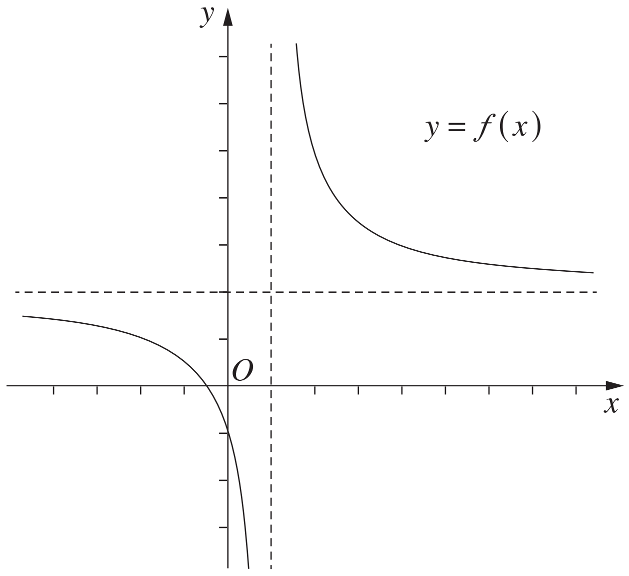
A. $\dfrac{23\pi}{12}$

B. $\dfrac{11\pi}{12}$

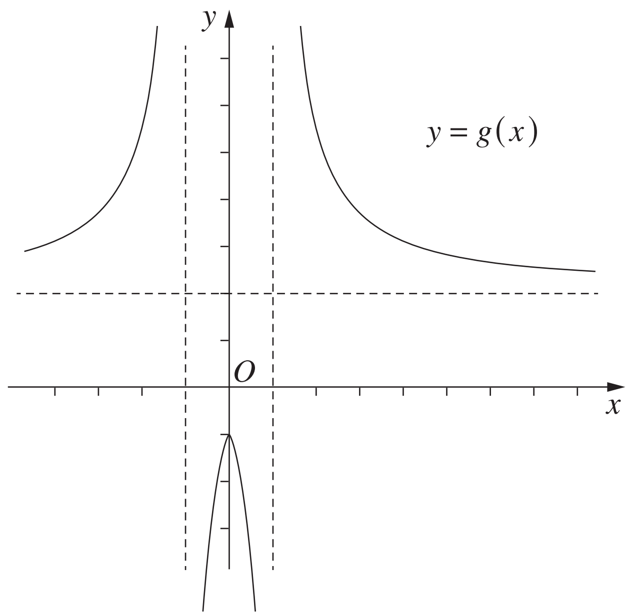
C. $\dfrac{\pi}{12}$

D. $-\dfrac{11\pi}{12}$

2 The graph of $f(x) = \dfrac{3}{x-1} + 2$ is shown.



The graph of $f(x)$ was transformed to get the graph of $g(x)$ as shown.



What transformation was applied?

A. $g(x) = f(|x|)$

B. $g(x) = \sqrt{f(x)}$

C. $g(x) = f(-x)$

D. $g(x) = \dfrac{1}{f(x)}$

3 Let $P(x)$ be a polynomial of degree 5. When $P(x)$ is divided by the polynomial $Q(x)$, the remainder is $2x + 5$.

Which of the following is true about the degree of Q?

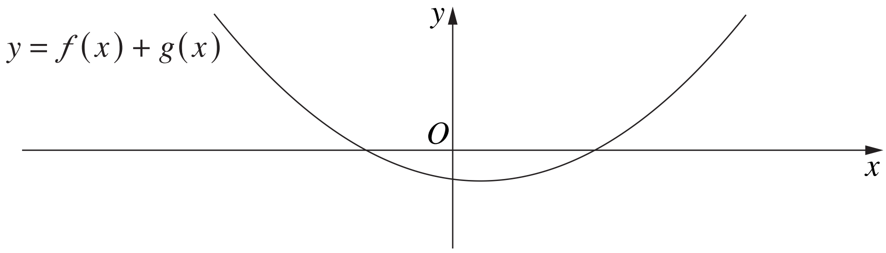
A. The degree must be 1.

B. The degree could be 1.

C. The degree must be 2.

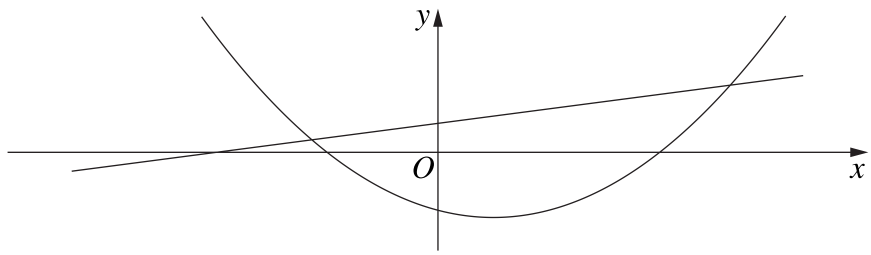
D. The degree could be 2.

4 The diagram shows the graph of the sum of the functions $f(x)$ and $g(x)$.

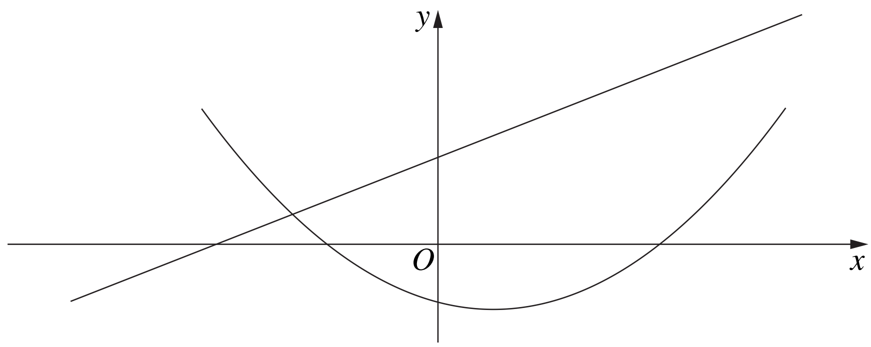


Which of the following best represents the graphs of both $f(x)$ and $g(x)$?

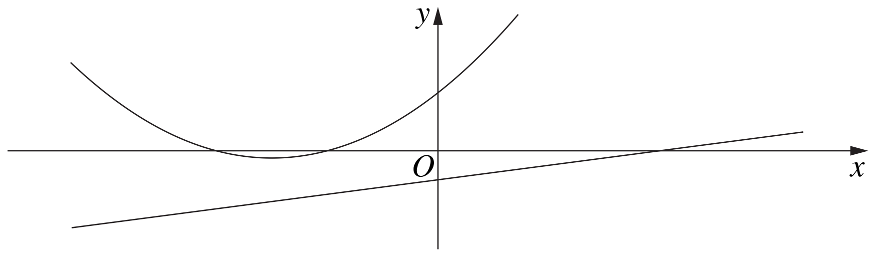
A.



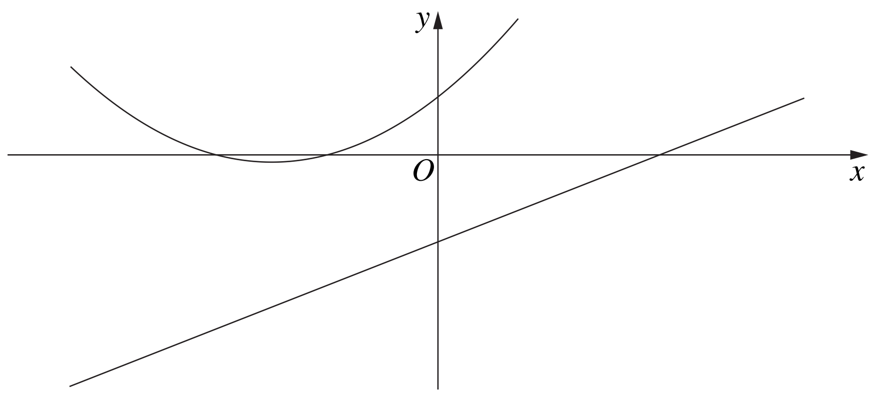
B.



C.



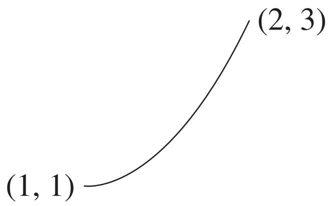
D.



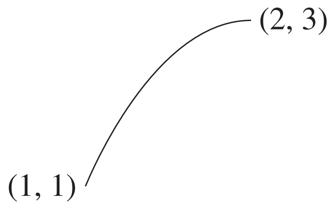
5 A curve is defined in parametric form by $x = 2 + t$ and $y = 3 - 2t^2$ for $-1 \le t \le 0$.

Which diagram best represents this curve?

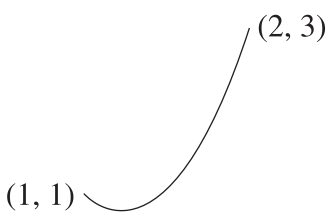
A.



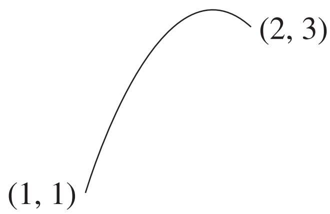
B.



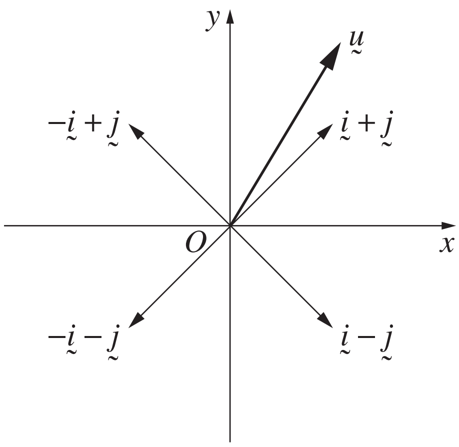
C.



D.



6 The following diagram shows the vector $\underset{\sim}{u}$ and the vectors $\underset{\sim}{i} + \underset{\sim}{j}$, $-\underset{\sim}{i} + \underset{\sim}{j}$, $-\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{i} - \underset{\sim}{j}$.



DRAWN
TO SCALE

Which statement regarding this diagram could be true?

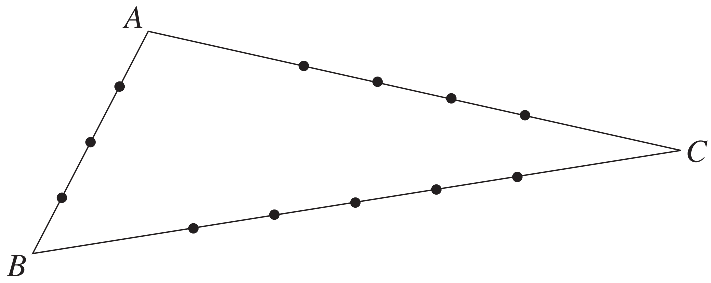
A. The projection of $\underset{\sim}{u}$ onto $\underset{\sim}{i} + \underset{\sim}{j}$ is the vector $1.1\underset{\sim}{i} + 1.8\underset{\sim}{j}$.

B. The projection of $\underset{\sim}{u}$ onto $-\underset{\sim}{i} + \underset{\sim}{j}$ is the vector $-0.4\underset{\sim}{i} + 0.4\underset{\sim}{j}$.

C. The projection of $\underset{\sim}{u}$ onto $-\underset{\sim}{i} - \underset{\sim}{j}$ is the vector $3.2\underset{\sim}{i} + 3.2\underset{\sim}{j}$.

D. The projection of $\underset{\sim}{u}$ onto $\underset{\sim}{i} - \underset{\sim}{j}$ is the vector $0.5\underset{\sim}{i} - 0.5\underset{\sim}{j}$.

7 The diagram shows triangle *ABC* with points chosen on each of the sides. On side *AB*, 3 points are chosen. On side *AC*, 4 points are chosen. On side *BC*, 5 points are chosen.



How many triangles can be formed using the chosen points as vertices?

A. 60

B. 145

C. 205

D. 220

8 The angle between two unit vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is θ and $|\underset{\sim}{a} + \underset{\sim}{b}| < 1$.

Which of the following best describes the possible range of values of θ?

A. $0 \le \theta < \frac{\pi}{3}$

B. $0 \le \theta < \frac{2\pi}{3}$

C. $\frac{\pi}{3} < \theta \le \pi$

D. $\frac{2\pi}{3} < \theta \le \pi$

9 A given function $f(x)$ has an inverse $f^{-1}(x)$.

The derivatives of $f(x)$ and $f^{-1}(x)$ exist for all real numbers x.

The graphs $y = f(x)$ and $y = f^{-1}(x)$ have at least one point of intersection.

Which statement is true for all points of intersection of these graphs?

A. All points of intersection lie on the line $y = x$.

B. None of the points of intersection lie on the line $y = x$.

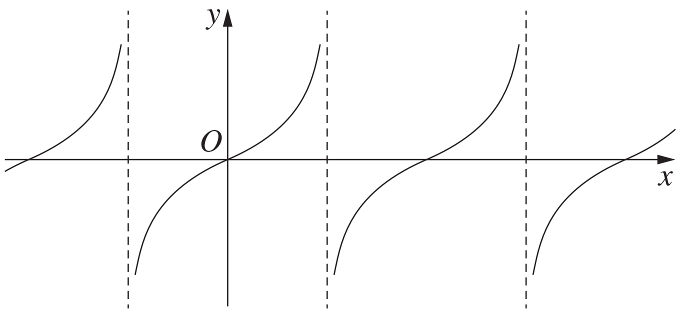
C. At no point of intersection are the tangents to the graphs parallel.

D. At no point of intersection are the tangents to the graphs perpendicular.

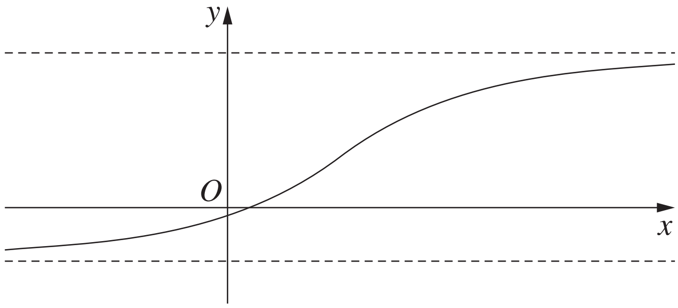
10 Which of the following could be the graph of a solution to the differential equation

$$\frac{dy}{dx} = \sin y + 1?$$

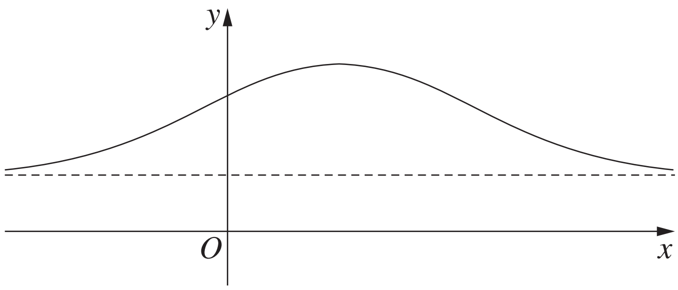
A.



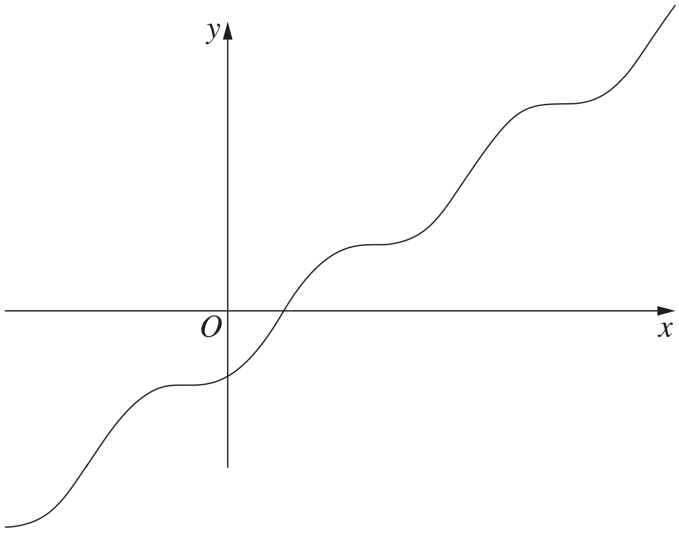
B.



C.



D.



Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet

(a) For the vectors $\underset{\sim}{u} = \underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} + \underset{\sim}{j}$, evaluate each of the following.

(i) $\underset{\sim}{u} + 3\underset{\sim}{v}$ **1**

(ii) $\underset{\sim}{u} \cdot \underset{\sim}{v}$ **1**

(b) Find the exact value of $\int_0^1 \frac{x}{\sqrt{x^2+4}}\,dx$ using the substitution $u = x^2 + 4$. **3**

(c) Find the coefficients of x^2 and x^3 in the expansion of $\left(1 - \frac{x}{2}\right)^8$. **2**

(d) The vectors $\underset{\sim}{u} = \begin{pmatrix} a \\ 2 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} a-7 \\ 4a-1 \end{pmatrix}$ are perpendicular. **2**

What are the possible values of a?

(e) Express $\sqrt{3}\sin(x) - 3\cos(x)$ in the form $R\sin(x + \alpha)$. **3**

(f) Solve $\frac{x}{2-x} \geq 5$. **3**

Question 12 (16 marks) Use the Question 12 Writing Booklet

(a) A direction field is to be drawn for the differential equation **2**

$$\frac{dy}{dx} = \frac{x - 2y}{x^2 + y^2}.$$

On the diagram on page 1 of the Question 12 Writing Booklet, clearly draw the correct slopes of the direction field at the points P, Q and R.

(b) A sports association manages 13 junior teams. It decides to check the age of all players. Any team that has more than 3 players above the age limit will be penalised. **2**

A total of 41 players are found to be above the age limit.

Will any team be penalised? Justify your answer.

(c) Find the equation of the tangent to the curve $y = x \arctan(x)$ at the point with coordinates $\left(1, \frac{\pi}{4}\right)$. Give your answer in the form $y = mx + c$. **3**

Question 12 continues on page 12

(d) In a room with temperature 12°C, coffee is poured into a cup. The temperature of the coffee when it is poured into the cup is 92°C, and it is far too hot to drink.

The temperature, T, in degrees Celsius, of the coffee, t minutes after it is made, can be modelled using the differential equation $\frac{dT}{dt} = k\left(T - T_1\right)$, where k is the constant of proportionality and T_1 is a constant.

(i) It takes 5 minutes for the coffee to cool to a temperature of 76°C. **3**

Using separation of variables, solve the given differential equation to show that $T = 12 + 80e^{\frac{t}{5}\ln\left(\frac{4}{5}\right)}$.

(ii) The optimal drinking temperature for a hot beverage is 57°C. **1**

Find the value of t when the coffee reaches this temperature, giving your answer to the nearest minute.

(e) A game consists of randomly selecting 4 balls from a bag. After each ball is selected it is replaced in the bag. The bag contains 3 red balls and 7 green balls. For each red ball selected, 10 points are earned and for each green ball selected, 5 points are deducted. For instance, if a player picks 3 red balls and 1 green ball, the score will be $3 \times 10 - 1 \times 5 = 25$ points. **2**

What is the expected score in the game?

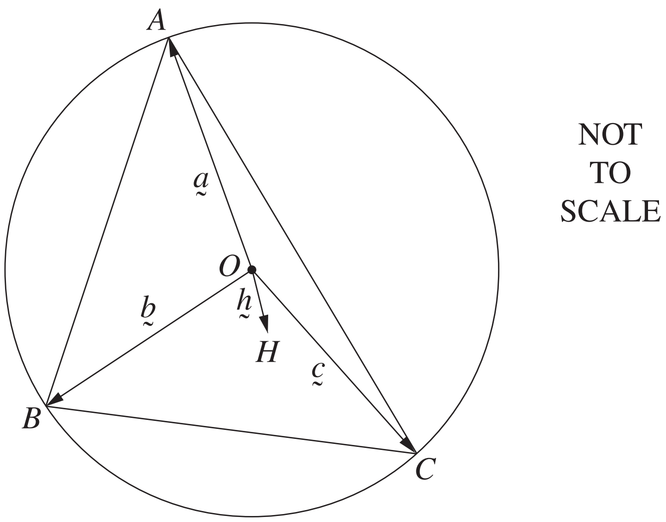
(f) Use mathematical induction to prove that $15^n + 6^{2n+1}$ is divisible by 7 for all integers $n \geq 0$. **3**

End of Question 12

Question 13 (14 marks) Use the Question 13 Writing Booklet

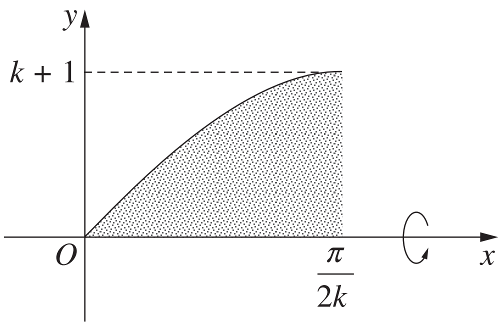
(a) Three different points A, B and C are chosen on a circle centred at O. **3**

Let $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$ and $\underset{\sim}{c} = \overrightarrow{OC}$. Let $\underset{\sim}{h} = \underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}$ and let H be the point such that $\overrightarrow{OH} = \underset{\sim}{h}$, as shown in the diagram.



Show that $\overrightarrow{BH}$ and $\overrightarrow{CA}$ are perpendicular.

(b) A solid of revolution is to be found by rotating the region bounded by the x-axis and the curve $y = (k+1)\sin(kx)$, where $k > 0$, between $x = 0$ and $x = \frac{\pi}{2k}$ about the x-axis. **3**



Find the value of k for which the volume is π^2.

Question 13 continues on page 14

Question 13 (continued)

(c) The function f is defined by $f(x) = \sin(x)$ for all real numbers x. Let g be the function defined on $[-1, 1]$ by $g(x) = \arcsin(x)$. **2**

Is g the inverse of f? Justify your answer.

(d) The monic polynomial, P, has degree 3 and roots α, β, γ. **3**

It is given that

$$\alpha^2 + \beta^2 + \gamma^2 = 85 \text{ and}$$
$$P'(\alpha) + P'(\beta) + P'(\gamma) = 87.$$

Find $\alpha\beta + \beta\gamma + \gamma\alpha$.

(e) *You may use the information on page 18 to answer this question.*

A chocolate factory sells 150-gram chocolate bars. There has been a complaint that the bars actually weigh less than 150 grams, so a team of inspectors was sent to the factory to check. They randomly selected 16 bars, weighed them and noted that 8 bars weighed less than 150 grams.

The factory manager claims 80% of the chocolate bars produced by the factory weigh 150 grams or more.

(i) The inspectors used the normal approximation to the binomial distribution to calculate the probability, $\mathcal{P}$, of having at least 8 bars weighing less than 150 grams in a random sample of 16, assuming the factory manager's claim is correct. **2**

Calculate the value of $\mathcal{P}$.

(ii) The factory manager disagrees with the method used by the inspectors as described in part (i). **1**

Explain why the method used by the inspectors might not be valid.

End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet

(a) Find the particular solution to the differential equation $(x-2)\dfrac{dy}{dx} = xy$ that passes through the point $(0, 1)$. **4**

(b) The vectors $\vec{u}$ and $\vec{v}$ are not parallel. The vector $\vec{p}$ is the projection of $\vec{u}$ onto the vector $\vec{v}$. **3**

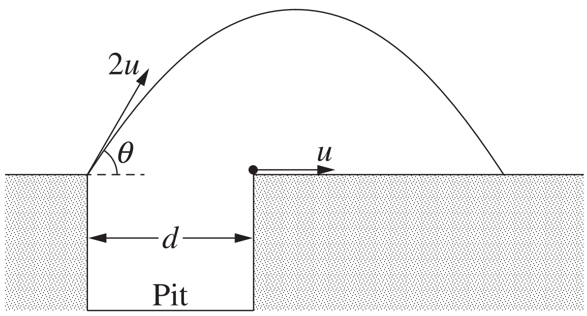
The vector $\vec{p}$ is parallel to $\vec{v}$ so it can be written $\lambda_0\vec{v}$ for some real number λ_0. (Do NOT prove this.)

Prove that $|\vec{u} - \lambda\vec{v}|$ is smallest when $\lambda = \lambda_0$ by showing that, for all real numbers λ, $|\vec{u} - \lambda_0\vec{v}| \le |\vec{u} - \lambda\vec{v}|$.

Question 14 continues on page 16

(c) A video game designer wants to include an obstacle in the game they are developing. The player will reach one side of a pit and must shoot a projectile to hit a target on the other side of the pit in order to be able to cross. However, the instant the player shoots, the target begins to move away from the player at a constant speed that is half the initial speed of the projectile shot by the player, as shown in the diagram below. **4**

The initial distance between the player and the target is d, the initial speed of the projectile is $2u$ and it is launched at an angle of θ to the horizontal. The acceleration due to gravity is g. The launch angle is the ONLY parameter that the player can change.



Taking the position of the player when the projectile is launched as the origin, the positions of the projectile and target at time t after the projectile is launched are as follows.

$$\vec{r}_P = \begin{pmatrix} 2ut\cos\theta \\ 2ut\sin\theta - \frac{g}{2}t^2 \end{pmatrix} \quad \text{Projectile}$$

$$\vec{r}_T = \begin{pmatrix} d + ut \\ 0 \end{pmatrix} \quad \text{Target}$$

(Do NOT prove these.)

Show that, for the player to have a chance of hitting the target, d must be less than 37% of the maximum possible range of the projectile (to 2 significant figures).

Question 14 continues on page 17

(d) *You may use the information on page 18 to answer this question.* **4**

An airline company that has empty seats on a flight is not maximising its profit.

An airline company has found that there is a probability of 5% that a passenger books a flight but misses it. The management of the airline company decides to allow for overbooking, which means selling more tickets than the number of seats available on each flight.

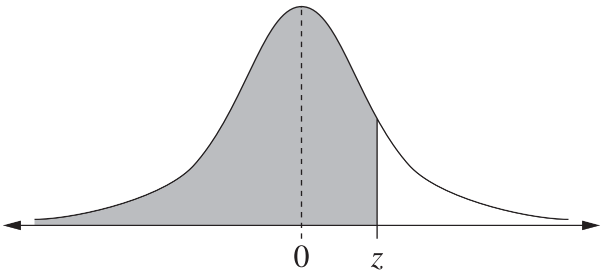
To protect their reputation, management makes the decision that no more than 1% of their flights should have more passengers showing up for the flight than available seats.

Given management's decision and using a suitable approximation, find the maximum number of tickets that can be sold for a flight which has 350 seats.

End of paper

You may use the information below to answer Question 13 (e) and Question 14 (d).

Table of values $P(Z \le z)$ for the normal distribution $N(0, 1)$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995

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NSW GOVERNMENT

NSW Education Standards Authority

Centre Number

Student Number

2022 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Extension 1

Writing Booklet

Question 12

Instructions

- Use this Writing Booklet to answer Question 12.
- Write the number of this booklet and the total number of booklets that you have used for this question (eg: 1 of 3).
- Write your Centre Number and Student Number at the top of this page.
- Write using black pen.
- You may ask for an extra writing booklet if you need more space.
- If you have not attempted the question(s), you must still hand in the writing booklet, with 'NOT ATTEMPTED' written clearly on the front cover.
- You may NOT take any writing booklets, used or unused, from the examination room.

12

[ ] of [ ]

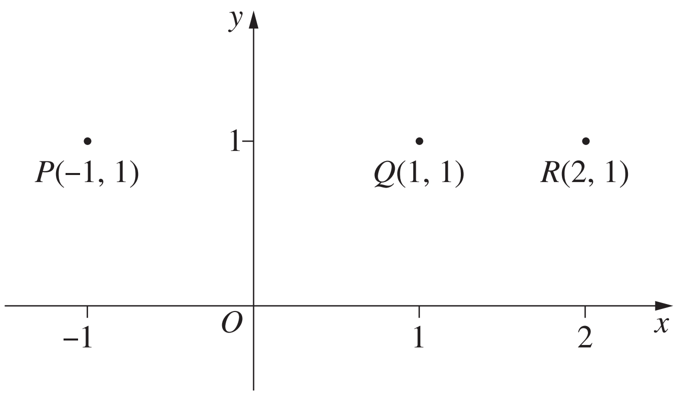
this booklet — number of booklets for this question

Start here for Question Number: **12**

(a) A direction field is to be drawn for the differential equation

$$\frac{dy}{dx} = \frac{x - 2y}{x^2 + y^2}.$$

Clearly draw the correct slopes of the direction field at the points P, Q and R shown below.



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NSW GOVERNMENT

NSW Education Standards Authority

2022 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

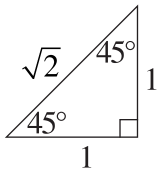
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

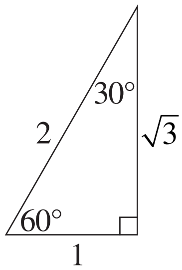


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

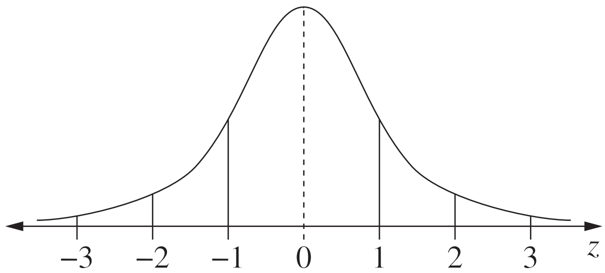
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$